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ANALYZING RISK IN AGROFORESTRY SYSTEMS USING A PORTFOLIO APPROACH

A Case Study from the United Kingdom

1. INTRODUCTION

Sanchez (1995) chose to open his well-known paper in the journal *Agroforestry Systems* with a definition of agroforestry as “the traditional practice of growing trees on farms for the benefit of the farm family”. Although not explicitly stating agroforestry is a discipline focused mainly on developing countries, it is easy to make such an inference. This poses a problem that is central to this chapter. The design, benefits and uptake of agroforestry systems depend, in part, on risk. However, models that deal with risk are intensely “data hungry”. For example, in response to the 1989 question, “does agroforestry pay?” it was stated that agroforesters could produce “little concrete data by way of an answer” (“Does Agroforestry Pay”, 1989). Similar comments have been made more recently (Scherr & Current, 1997) but, unfortunately, the data requirements of a full risk analysis are far in excess of those for straightforward profitability calculations and beyond any reasonable expectations of the data that might come to hand in the tropical/subtropical arena.

This is even true of the case study chosen here, a silvopastoral system in the United Kingdom. Even after extensive research, the data for a full risk analysis are not available. Furthermore, in the UK it can be assumed that crops are sold in markets and not used for subsistence or household needs thus allowing the analysis to proceed in terms of well-defined monetary values. A somewhat stronger assumption is that the farmer has access to perfect capital markets. This is, of course, unlikely to be the case in many applications of agroforestry when the absence of capital markets, or indeed markets in general, mean that farmers will put a premium on the immediate output of crops, often to the disadvantage of tree-based

elements of the system (Izac, 2003). Therefore, the main part of the chapter will deal with a temperate agroforestry system, thus permitting many simplifying assumptions. In the conclusions to the chapter a number of points will be made that deal with extending the method outlined here to what might be thought to be more “typical”, tropical agroforestry situations.

The model of risk analysis to be applied here is portfolio theory. This is a theory that rigorously defines the concept of diversification in the context of reducing risk in financial portfolios. At the core of the theory lies the correlation of the returns of the various financial assets constituting the portfolio. In essence, if two stocks are negatively correlated, the risk of a portfolio of both of them is less than that of the individual stocks.

This theory is thought to be of relevance to agroforestry for two reasons. First, the risk of an agroforestry system is often alluded to in the literature, especially in the context of such systems being desirable from the farmer’s point of view. Secondly, even if the economic aspects of diversification are not considered, the biological side of agroforestry systems is often described in terms of plant/animal interactions in a way that is reminiscent of portfolio theory.

In this chapter, the concepts of portfolio theory will be extended to cover the situation of agroforestry and it will be shown that the theory can offer important insights into the evaluation and design of agroforestry systems. The desirability of agroforestry as opposed to what Price (1995) referred to as “coarse-level” mixing, or simply growing crops on different parts of the farm, can be determined using a portfolio approach. It will also be seen that what is sometimes claimed to be a benefit of agroforestry – the crop interaction – can be a disadvantage.

However, as portfolio theory is likely to be an area with which those working in agroforestry are unfamiliar, a brief introduction follows.

2. PORTFOLIO THEORY AND RISK ANALYSIS

The hallmark of economic theory over the last fifty years or so has been the widespread introduction of risk analysis. Nowhere has this been more true than in the area of financial markets. A landmark work in this field is Markowitz’s (1959) portfolio analysis in which the principles of reducing the risk of a portfolio of stocks were outlined (for a good overview of the area in the context of finance see Elton & Gruber, 1995).

2.1. *The basic model*

Markowitz argued that investors were interested in the overall return and risk of their portfolio and not, directly, the returns and risk of the individual stocks. In the simple two-asset case Markowitz’s model takes the following form. Let w_1 be the proportion of the total portfolio invested in asset one and w_2 be the proportion invested in asset two. If the return on asset one is R_1 and that for asset two is R_2 the return on the portfolio R_p will be given by equation 1.

$$R_p = w_1 R_1 + w_2 R_2 \quad (1)$$

The contribution of Markowitz was in introducing what is a simple piece of statistics into finance; the definition of the risk of the portfolio.

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{1,2} \quad (2)$$

Here, σ_1 represents the standard deviation of the returns of asset one and $\sigma_{1,2}$ is the covariance of returns of the two assets. This term can be replaced by $\sigma_1 \sigma_2 \rho_{1,2}$, where $\rho_{1,2}$ is the correlation of the returns R_1 and R_2 .

This correlation of returns is the core of Markowitz's ideas. Unless the correlation of stock returns is perfect and positive (i.e., unless $\rho_{1,2} = +1$) there will always be a risk reducing effect to diversification. Indeed, if there is perfect negative correlation it is easy to show that a portfolio with asset one in the proportion $\sigma_2/\sigma_1 + \sigma_2$ will have a risk of zero even though its individual constituents are, in isolation, risky. Indeed, this has long been known in agricultural markets where the use of futures to hedge against price fluctuations is merely an application of this principle.

An insight from portfolio theory that seems a little counterintuitive is the role of currency markets in international diversification. While there are many aspects to the production of crops for international markets, it is likely that the addition of the exchange rate into the equation will actually reduce risk for farmers. As long as the output of crops is not perfectly positively correlated with exchange rate fluctuations (which is likely to be the case) the revenues from the sale of crops on the international market expressed in the local currency will be somewhat more stable as the result of the less-than-perfect correlation of crop yields and currency movements. Indeed, in the analysis of diversification of stock portfolios it is usually found that international diversification reduces risk *because* of the riskiness of the currency markets (see Elton & Gruber, 1995 for references on this topic).

2.2. Larger portfolios

The main part of this chapter deals with what, on the face of it, is a two-asset case: a sheep/tree silvopastoral system. However, as a forty-year rotation is used, the system is, from a portfolio point of view, a forty-one asset system – forty lamb crops and one timber crop. So a generalization of the equations above into an n -asset system is needed. This is shown in equations 3 and 4.

$$R_p = \sum_{i=1}^{i=n} w_i R_i \quad (3)$$

Table 1. Variance/covariances, correlations, and expected net present value (ENPV) for a hypothetical three-crop system.

| Crop | Variance/covariance | | | Correlation | | ENPV |
|------|---------------------|------|----|-------------|---|------|
| | A | B | C | B | C | |
| A | 144 | 64.8 | 0 | 0.6 | 0 | 100 |
| B | | 81 | 0 | | 0 | 75 |
| C | | | 64 | | | 50 |

$$\sigma_p^2 = \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} w_i w_j \sigma_{i,j}$$

(4)

In Blandon (1985) a hypothetical three-crop system with the characteristics described in Table 1 was examined.

The figures in Table 1 give rise to Figure 1. On the y-axis the expected net present value (ENPV) of the agroforestry system is given. On the x-axis the risk is measured as the standard deviation of the NPV. The points at the end of the line in the diagram, labeled A and C, represent the risk and return from crops A and C respectively if they are grown as monocultures (i.e., $w_a=1$ or $w_c=1$). The line joining these two points represents mixtures of the crops that minimize risk for any given ENPV.

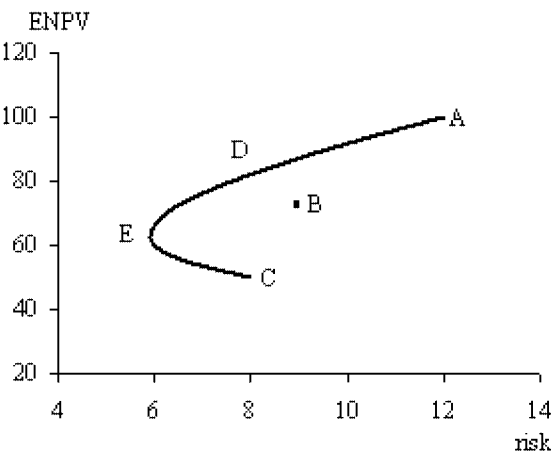


Figure 1. Risk analysis for a hypothetical three-crop system.

The mathematics and logic of the model ensure that the line will be weakly convex from below. In the extreme case, in which there is perfect positive correlation between A and C, the line joining points A and C would be straight. When correlation is less than plus one, there are some benefits from diversification and so the ENPV obtained from a crop combination will have less risk than that which would be implied by points on a straight line.

It is important to note that is point B, representing a pure crop B, is not a rational choice. It is dominated by points along the line CA, meaning that there are combinations of crops that give a higher return for less risk. For example, point D represents a combination of 59.3% of crop A, 21.5% of crop B and 19.3% of crop C (with rounding errors). This gives a return of 85 and a risk of 8.55 as a standard deviation or a variance of 73.2, better than B.

The line CA “bends back” on itself. Point E represents a minimum risk agroforestry system. The point actually represents 8% crop A, 37% crop B and 55% crop C. The segment of the line EA is known as the *efficient frontier* as it shows the combination of crops that give the maximum return for any given level of risk and would be the set from which any rational, risk-averse farmer would choose. The upshot of this is that the only rational monoculture is crop A. All other monocultures are bettered by some agroforestry system on the segment from E to A.

Note that this bending back does not always occur. It is caused here by the lack of correlation between two of the crops.

3. PORTFOLIO THEORY AND AGROFORESTRY

It is often stated that one of the benefits of agroforestry lies in the ability of different crops to exploit different aspects of the resources available. Early writings tended to concentrate on the competition for moisture and the combination of crops with different rooting characteristics. Similarly, silvopastoral or silvoarable systems emphasize the complementary nature of the crops, in the case of the sheep to be looked at here, the shading and shelter provided by the trees is said to increase the conversion of pasture into body mass and pasture growth is aided by trees in the early stages of the rotation. This is reminiscent of portfolio theory with the correlation of output substituted for the correlation of price. However, there are some significant differences between finance and agroforestry that make it impossible to transplant Markowitz’s ideas into agroforestry without some modification.

3.1. Returns and profits (the expected values)

The model developed by Markowitz deals with the single variable of returns but, in agroforestry, it is likely that profits are the relevant decision variable. However, profit consists of prices multiplied by volumes, both of which might be considered stochastic in nature. Both Blandon (1985) and Lilieholm and Reeves (1991) chose net present value as the basic stochastic variable and so ignored the sources of

correlation between the profits of two crops. Correlation can come from price interaction or volume interaction. Thus, two threads can be identified; price interrelationships, the basis of portfolio theory in finance, and volume interrelationships, which have been the main thrust in agroforestry. This chapter attempts explicitly to combine these two threads.

Similarly, there might be correlations between the price and volume of a single crop. For example, high prices might call forth more output; alternatively, high output might depress prices. Having said this, however, for simplicity it will be assumed in all of the following that the correlation between price and volume *for a given crop* is zero.

The explicit introduction of volume interactions means that the proportions of the assets, w_i , lose their meaning. As Price (1995) points out, the diversification possibilities offered by price variation alone are available by “coarse-scale mixing of *separate* single-product systems under one ownership”, but most of the work in agroforestry has been directed at finding systems where the output is increased relative to that obtained at the coarse level. This has been quantified in competition studies by the concept of the *land equivalent ratio* (LER) (Mead & Willey, 1980; Moseley, 1994) shown in equation 5.

$$LER = \frac{Y_1}{M_1} + \frac{Y_2}{M_2} \quad (5)$$

Here, Y_1 shows the physical yield of crop 1 when grown in some form of mixture and M_1 shows the crop's yield on the same area when grown as a monoculture. The LER shows the relative land area that would be required to grow the same total volume of the two crops that are produced in an intimate mixture of some sort. So, a land equivalent ratio of 1.2 indicates that 20% more land would be required to produce the same volume of crops in a coarse-level mixture. In coarse-level mixing it is clear that the elements of the equation, Y_i/M_i would all be equal to the proportion of the total farm area devoted to the crops and would analogous to w_i .

But it is clear the land equivalent ratio describes only one part of the story. It is not true to state, as does Moseley (1994) that “a result of 0.91 ... suggests that a farmer would not choose the agroforestry system over the monoculture”. If the reduction in output is more than compensated for by a reduction in risk, agroforestry could be indicated.

Logically, of course, the cost elements should also be considered to be stochastic in nature and this would introduce a further set of correlations into the system. Here, for simplicity, costs will be considered to be non-stochastic.

3.2. Sequential Agroforestry Systems (the variances and covariances)

Using the terminology of Sanchez's (1995) paper, most agroforestry systems are sequential. Thus, for the derivation of the overall profit of the system, a common

time frame is needed. In the system looked at below, a 40 year rotation period is chosen. The investment becomes, therefore, a 41-asset portfolio. There are forty outputs from the agricultural component of the system and one from the forestry part.

While a 41-asset portfolio can easily be handled by equations 3 and 4, the introduction of more than one time period also introduces the possibility of serial correlation in the volume and price figures. For reasons outlined below, these correlations can be taken to be zero.

4. COARSE-LEVEL MIXING IN A TWO-CROP SYSTEM

The following three sections look at a two-crop system, ash (*Fraxinus excelsior*) and rye-grass pasture supporting sheep. The outputs of the system are lambs and ash timber for joinery. Most of the figures relate to experiments in the system undertaken in the United Kingdom and specifically the experimental sites in North Wales. The basic example closely follows the figures in Thomas and Willis (2000).

The overall objective is the calculation of two efficient frontiers. The first, to be considered in this section, is the simple “coarse-level mixing” frontier where it is assumed that the crops are grown in spatially distinct areas. The only difficulty here lies in the derivation of the variance and covariance terms for net present values derived from two stochastic variables. The second efficient frontier, to be examined in the next section, is the agroforestry one in which the crops are assumed to be intimately mixed and interactions are taken into account.

In terms of the points on Figure 1, these two efficient frontiers will have the same end-points, referring to monocultures. It is useful, therefore, to look at the end-point solutions first after discussing the basic approach to the price and volume variables.

4.1. Price and cost variables

The prices of timber and sheep are treated in a specific way, based on the *efficient market hypothesis* from financial economics (see Elton & Gruber, 1995 for a description in finance). Under this hypothesis price changes follow a geometric Brownian motion that can be modeled as follows. Let the price in period t be P_t and let the logarithm of the proportional change in price or log price-relative, $\ln(P_t/P_{t-1})$, be an independent drawing from a normally distributed random variable with a mean of μ and a variance of σ^2 . The implication of such a structure is that the expected price in the next period is the price now with the growth factor μ included. Thus, there is no way to predict the price in the future other than extrapolation using the growth rate μ . Any information regarding the market in question and ideas of future scarcity or glut is already factored into the current price. The expected price in period t , $E(P_t)$ and the variance of price are given by the equations below where P_1 is the current, known price.

$$E(P_t) = P_1 e^{\mu t} \tag{6}$$

$$\sigma_t^2 = P_1^2 e^{2\mu t} \left(e^{t\sigma^2} - 1 \right) \tag{7}$$

The variance of the price in the future increases, reflecting the idea that periods in the more distant future are less certain. Such a model of prices has the advantage that it strips out any serial correlation and reduces the data requirements of the model. Another advantage of the model is that complications such as storing in times of historically low prices in the expectation of a price rise can be ignored – a complication in the derivation of a rotation length. It has the deficiency that it seems quite reasonable to assume that a good year for lamb birth and survival could depress price in the current year and so would tend to lead to the expectation of negative serial correlation. Increased storage of meat, for example, would go a long way to weaken such linkages. The actual figures employed are presented in the following table and their derivation is explained below.

The figures for sheep are taken from the annual *Farm Business Survey in Wales* (Institute of Rural Studies, various years). This survey is undertaken by the Institute of Rural Studies at the University of Wales, Aberystwyth. The figures from 1985 to 2001 for all flock sizes on Welsh lowland farms show an average lamb price of £35.29. The higher figure used here reflects the extra income earned by farmers in the form of sale of culled ewes, fleeces and the subsidy received. The figure in Table 2 is, more correctly, gross income. The variance of the log-price relative was calculated on the basis of lamb prices alone, the implication being that the other income elements were fixed.

Timber prices are more problematical. In Britain, the market for hardwood timber has tended to be scattered and so there are relatively few published statistics of sufficient length or consistency to allow estimates of time trends or variances. So the following approach was adopted. The Forestry Commission in the United Kingdom publishes a survey of coniferous standing timber prices and these it converts into a Laspere index. The conversion accounts for the change in the composition of the timber offered for sale by the Commission. The variance of the log-price relatives of this index was found and, as it represents the variance of a

Table 2. The price variables in the model.

| | Lamb data | Timber data |
|---------------------------------------|-----------------|-------------------|
| Current price P_t | £46.97 a head | £50 a cubic meter |
| Average annual growth, μ | 0.41% | 2.9% |
| Variance of annual growth, σ^2 | 0.037 | 0.015 |
| Cost | £16.96 per lamb | £1.16 per tree |

growth term, it was assumed that ash prices showed the same variance of growth. The price now, P_t , is in the range, relative to coniferous prices, suggested by the Forestry Commission (Insley, 1988). Whilst grants might be available for the forestry part of the investment, depending on the planting density, they will not be included in the analysis here.

The implication for prices in this model is shown in Figure 2. There the thicker line shows the expected price over the forty-year study period. The shaded areas show the 95% confidence intervals for the prices. The implication of the model for risk is clear.

The cost figures are adapted from Thomas and Willis (2000) and the Aberystwyth surveys. Note that the sheep costs are on a per lamb basis while the forestry costs are on a per tree basis. This means that high fertility will lead to higher costs for sheep farming. In forestry, the costs of planting are spread over the years of the rotation and the figure in the table is the present value. It is deemed to be constant regardless of final output. This difference causes the equations that follow to have slightly different forms for sheep and forestry – see the Appendix.

4.2. Volume variables

The variance of lamb production was treated in a similar way to that of price. Proportional changes in fertility from year to year were deemed to be drawings from a static distribution with a mean of zero. Although it could be argued that fertility will increase over the study period, it was assumed here not to do so. The average fertility over the 1985 to 2001 period according to the Aberystwyth survey is 1.131

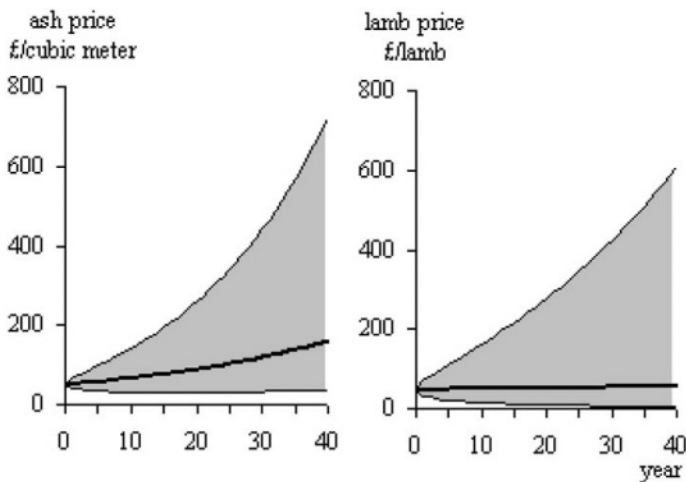


Figure 2. The expected prices and their 95% confidence intervals.

lambs per ewe with a variance of 0.0122. The stocking rate used for monoculture was 12 ewes per hectare. This is the same as Thomas and Willis (2000), similar to Willis, Thomas, and van Slycken (1993) but more than reported by Crowe and McAdam (1999).

For ash, the situation is less straightforward. The volume produced will depend on the rotation and here a forty-year rotation is taken (thereby ignoring the complications that the rotation will depend on the discount rate which will depend on risk). No agroforestry experiments in Britain have yet run to a full rotation and so, in most of the studies, it is assumed that the cumulative yield will be similar to that contained in the Forestry Commission's *Forest Management Tables* (Hamilton & Christie, 1971).

The *Forest Management Tables* were developed when monoculture was seen as being the only method that the Forestry Commission would use. Their underlying idea is that, within a wide range of stocking rates, the cumulative timber yield is constant. The tables are based on so-called *yield classes*, which are denoted by the maximum mean annual increment per hectare for a site. In Thomas and Willis (2000) an ash yield class of 12 is assumed. This generates a cumulative yield of 480 cubic meters per hectare over 40 years. Usually, a substantial portion of this would have been removed as thinnings. However, to keep the analysis here in line with the studies in Britain, it will be assumed that only 25% of the cumulative yield is removed in this way, giving an average final cut of 75% of cumulative yield, or 360 cubic meters. To simplify the analysis, and without too much loss of reality, it will be assumed that thinnings are removed and sold at cost so that the expected value of thinning (and the variance) is zero.

Very little on the variability of volume output is published, so the following approach was adopted. It is not unusual for yield classes to turn out to be one class higher or lower than estimated. In Britain, classes are denoted at intervals of two so it would be possible that the ash productivity was in yield class 10 or 14. However, the plantation thinning would allow adjustments to be made so that the final yield would be closer to the expected. Therefore, it is assumed that the final yield lies in the range of half this, from 440 to 520, and this is taken as being the 95% confidence interval of a normal distribution. The implication is that the variance of the yield is 416 which, when measured as a proportional factor, is 0.0018.

Table 3. The quantity variables in the model.

| | <i>Lamb data</i> | <i>Timber data</i> |
|-------------------|-------------------------|-------------------------|
| Stocking rate | 12 ewes/ha (N_s) | 1,800 stems/ha |
| Average fertility | 1.131 $E(F_s)$ | n/a |
| Variance | 0.0122 (σ_s^2) | 0.0147 (σ_t^2) |

4.3. Expected net present values

The expected net present value for forestry is calculated using the formula below where R is the rotation length of 40 years. The discount rate, r , is 5%. The cost, C_f varies according to the number of plants used but is constant for any given planting level. The subscript f refers to forestry and the expected price $E(P_{f,R})$ to the expectation of the price of timber in period R , the expectation being formed now. The expected revenue figure is just the expected price $E(P_{f,R})$ multiplied by expected quantity $E(V_f)$ - see the Appendix.

$$E(NPV_f) = \frac{E(P_{f,R})E(V_f)}{(1+r)^R} - C_f \quad (8)$$

The expected price of timber in year 40 is £160.47 and the resulting expected net present value is £6,119 per hectare.

The expected net present value of sheep production is merely the sum of the expected net present values for each period given by equation 9 below.

$$E(NPV_s) = \sum_{t=1}^{t=R} \frac{N_s E(F_s) \times (E(P_{s,t}) - C_s)}{(1+r)^t} \quad (9)$$

The result is £7,665 per hectare.

4.4. The risks

The risk from forestry derives from a single cash flow at the rotation end and this is given by equation 10 – see the Appendix. The variances in the square brackets are those of the growth rates and not those of absolute price or volume. The subscript pf refers to forestry price and vf to the volume output of forestry.

$$\sigma_f^2 = \frac{E(P_{f,R})^2 E(V_f)^2}{(1+r)^{2R}} [\sigma_{vf}^2 + R\sigma_{pf}^2 + R\sigma_{vf}^2 \sigma_{pf}^2] \quad (10)$$

Note that the variance of price is multiplied by R because of the assumption of timber prices being generated by a growth process. The variance of volume is not influenced in this way. The result is a risk, measured as a standard deviation, of 6,647.

The calculation is similar for sheep. The assumption of price and volume independence through time means that the net present values over the forty years are themselves independent random variables. Thus, the risk can be assumed to be the sum of the variances of the individual net present value terms. For each year the

risk, as defined by an equation similar to equation 10 – see Appendix – can be used and the terms summed. The resulting formula is given below. The difference in formulation results from the difference in the treatment of costs.

$$\sigma_s^2 = \sum_{t=1}^{t=R} \frac{E(V_s)^2}{(1+r)^{2t}} \left[E(P_{s,t})^2 (t\sigma_{ps}^2 + t\sigma_{vs}^2 \sigma_{ps}^2) + \sigma_{vs}^2 (E(P_{s,t}) - C_s^2) \right] \quad (11)$$

The result is a standard deviation of 1,306. Thus, sheep husbandry is considerably less risky than forestry. Although the summation of risks might suggest that the riskiness should increase, the volume of lamb production and the variance of early prices are much lower so the overall risk is reduced.

4.5. End-point solutions – monocultures compared

The results of the analysis are shown as points S (sheep) and F (forestry) in Figure 3. Any rational, risk-averse farmer would choose sheep husbandry over forestry as the expected net present value is greater and the risk is also less.

4.6. Coarse-level integration

The ability to mix trees and sheep on spatially separated parts of the farm means that these ENPVs in equations 8 and 9 can be mixed and any diversification deriving from price can be taken advantage of. However, none of the benefits of intimate

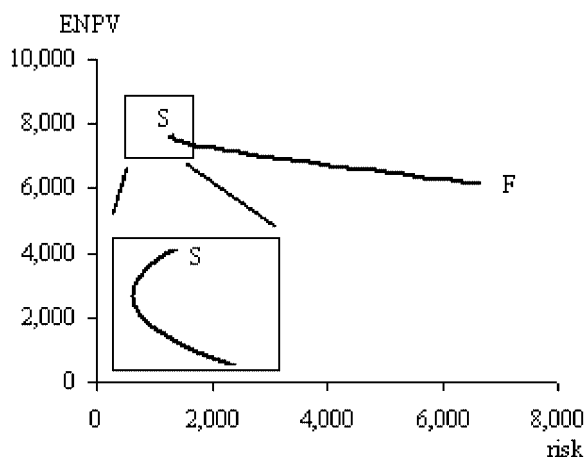


Figure 3. The efficient frontier with coarse-level mixing (box shows magnification of area around point S).

mixing of crops would be available. The relationship between risk and return depends upon the covariance of the expected net present values of forestry and agriculture. These are discussed in more detail in the following section so, as an initial assumption, assume that the price and volume of sheep and timber output are independent so that any covariance term is zero.

Under these assumptions the risk of the two crops together will be given by equation 4 where the covariance elements are zero, the variances are those calculated above and the weights w refer to the proportion of the total area of the farm unit under sheep or trees. The resulting frontier is shown in Figure 3 joining points S and F.

Although the line looks suspiciously linear, it does have a shape similar to that in Figure 1 but inverted. The curve enclosed by the box in Figure 3 shows a magnification of the area of the efficient frontier around point S. Moving from point S to point F involves, to begin with, a movement towards the left, in other words a reduction in risk. There is a combination of sheep and trees that minimizes risk and this is achieved by using 3.7% of the farm area for tree crops and the remaining 96.3% for sheep. This will give an expected net present value of £7,607 and a risk of 1,282.

The graph shows, therefore, that there are few benefits in diversification at the coarse level in this situation. The reasons for this will be discussed later, but first, it is necessary to look at the effect of a true agroforestry system and this means examining in some detail the interrelationships of the variables in the system.

5. A TRUE AGROFORESTRY SYSTEM

There are three sets of relevant figures if portfolio theory is to be applied to a true agroforestry system. The first set consists of the “agroforestry” relationships between the crops themselves. This is where most of the research has been undertaken and, in the terminology of portfolio theory, relates to the expected values of the system. The second is the set of variance terms on which virtually no work has been undertaken. The final set is the covariance element.

5.1. The expected values

The main input in this model, from which all other terms follow, is the initial stocking rate for trees. This ranges from 0 to 1,800. The latter is the stocking rate in the standard Forestry Commission yield tables and so can be seen as “conventional forestry”. Not many studies have looked at timber volume under the low stocking rates encountered in agroforestry so, here, the overall cumulative volume of the timber production in the agroforestry system is given by a Gompertz (double exponential) function (Cabernettes, Auclair, & Imam, 1999). This takes the form shown in Figure 4 with the volume per hectare being shown on the right-hand axis.

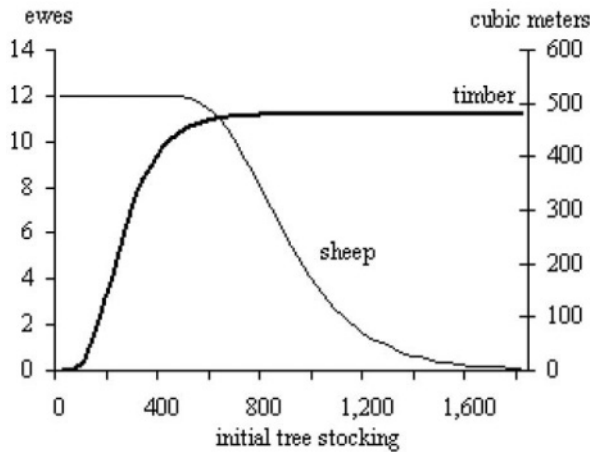


Figure 4. The Gompertz functions applied to timber production and ewe stocking.

The parameters of the function are shown in the Appendix. The rationale behind the curve is the same as that behind the Commission's yield tables. Over a wide range of stocking rates the cumulative output is the same, but at the bottom end the tree crop does not use all resources, so there is an output drop.

The diagram shows that at a stocking rate of 400 stems per hectare the cumulative output is equal to 419 cubic meters. This is less than the volume assumed by Thomas and Willis (2000) and compares with the 480 in the Commission's yield tables. The thinning yield is always 25% of final volume and, as stated earlier, it is assumed to be removed at cost with no stochastic elements and so is ignored in this model.

The stocking rates for ewes is also shown in Figure 4, with the initial flock size on the left-hand axis, the initial stocking rate depending on the tree stocking. The relationship is also modeled by a Gompertz function, and the parameters are included in the Appendix.

Thomas (1991) and Thomas and Willis (2000) hypothesize that there is a reduction in the production of pasture and, therefore, lamb production as the canopy closes. As none of the agroforestry experiments have been running long enough to calibrate this part of the system, Thomas and Willis (2000) use two possible decay functions. One reduces lamb yield by 10% the other by 25%. In both cases the lamb output begins to decline around year ten and stabilizes at the lower around year 24, similar in parts to the results of Teklehaimanot, Jones, and Sinclair (2002). In the model, as in Thomas and Willis's, the difference between the two assumptions is not great given the effect of discounting. Here a 25% figure was used with the parameters of the Gompertz function to generate the relationship shown in the Appendix.

The results of these assumptions are summarized in Figure 5 where the numbers in the graph refer to the initial tree stocking rates in terms of stems per hectare.

These figures are sufficient to produce a graph of the land equivalent ratio defined by equation 5 above. This is shown in Figure 6. The peak in the LER is at an initial planting rate of around 500 stems per hectare. Without an analysis of risk, therefore, the system of Thomas and Willis (2000) using a 400 stem per hectare planting rate would seem to be strong contender for the best agroforestry system.

5.2. The variance terms

There seems to be little or no work on the effect of the *variance* of crop output in agroforestry systems. All of the work is concentrated on finding the average or expected levels of outputs for different crop combinations. Mead and Willey (1980) refer to the “general belief that intercropping yields are more stable” but this, if true, could result from a number of possible combinations of variances and covariances. For example, if the outputs of two crops are independent the variance of their combined output will fall because of the zero covariance term in a suitably redefined equation 2. Indeed, a falling covariance could even be the result of negative correlation but increasing absolute variances.

In the absence of evidence to the contrary, therefore, it will be assumed that the variance terms in Tables 2 and 3 are unaltered under different crop combinations. As they take the form of proportional errors – see the Appendix – the implication is that a crop yield of, say, 5% above the expected is just as probable if the crop is grown alone as it would be if the crop was grown as a small part of an agroforestry system.

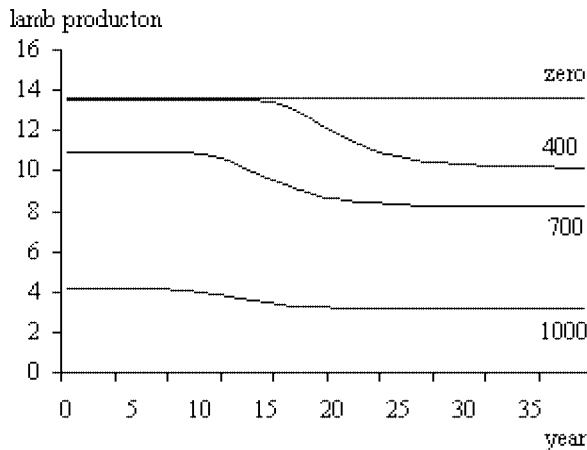


Figure 5. Lamb production under different tree stocking rates.

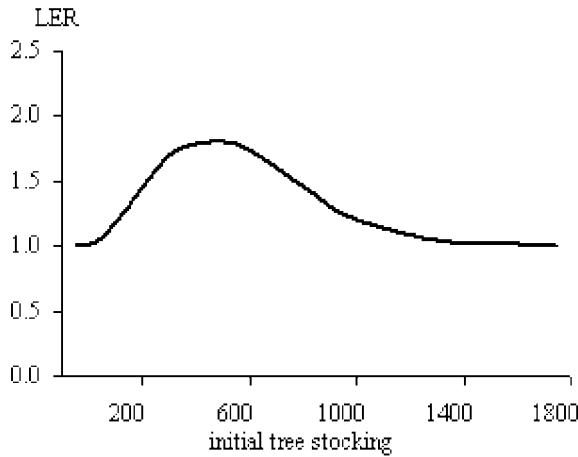


Figure 6. The land equivalent ratio under the assumptions here.

5.3. The covariance terms

The covariance of the net present values is the heart of the portfolio approach. However, the assumption of a lack of serial correlation between volumes and prices allows a great simplification. As there are 41 net present values in the model there should be 820 covariance terms. However, the only one that will be non-zero will be the covariance between the lamb output in year 40 and the timber output. If $\sigma_{vs,f}$ represents the covariance of the volumes of forestry and agriculture and $\sigma_{ps,f}$ the covariance of price, the covariance of the final NPVs is given below in equation 12.

$$\sigma_{s,f} = \frac{1}{(1+r)^{2R}} E(V_f)E(V_s)E(P_{f,R}) \left[(E(P_{s,R}) - C_s) \sigma_{vs,f} + E(P_{s,R}) \sigma_{ps,f} + E(P_{s,R}) \sigma_{vs,f} \sigma_{ps,f} \right] \quad (12)$$

The derivation of this unattractive formula is outlined in the Appendix.

5.4. The frontier

Under the assumptions above the frontier can be calculated and it is shown in Figure 7. Here it is assumed that all of the covariance terms in equation 12 are zero. This makes the frontier directly comparable with the one in Figure 3 reproduced as the thinner line.

The effects of agroforestry are to increase the ENPV but also to increase risk. The reduction in risk that occurs in financial applications of portfolio theory is, in fact, limited by the very nature of agroforestry. Half of one's investment in a financial asset halves one's returns and similarly reduces risk exposure. Half of

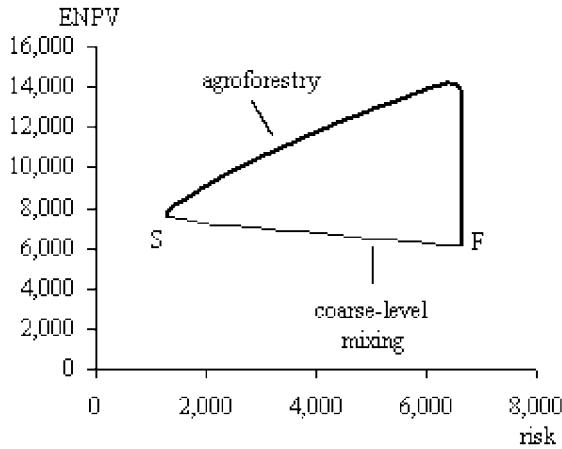


Figure 7. The efficient frontier for sheep/timber agroforestry systems (thicker line) and coarse-level mixing (thinner line).

one's land given over to one of the crops in an intimate mixture does not necessarily halve output and similarly, does not reduce the risk exposure in the same way. Hence, the benefits often ascribed, indeed desired, by agroforestry systems are, at best, a double-edged sword. The returns may well be enhanced, but the risk may also be increased.

The maximum ENPV is given by the peak of the frontier and corresponds to the 500 stems per hectare that maximized the LER in Figure 5. Note that this correspondence is not necessary. The LER is defined in terms of volumes while the ENPV is in value terms.

Recall that there was a slight movement to the 'left' of the coarse-level frontier and this was shown in Figure 3. But apart from this, every point on the coarse-level frontier shown by the thinner line in Figure 7 is dominated by points on the thicker line showing the agroforestry frontier. The implication is that, if mixing of sheep and trees is being considered, it is almost certainly better to mix them in the agroforestry sense than at the coarse-level.

5.5. The Sharpe index

In financial markets, the Sharpe index (SI) often used to measure portfolio performance is adapted for the situation here, and given in equation 13.

$$SI = \frac{E(NPV) - K}{\sigma} \quad (13)$$

The index shows the risk-adjusted, additional benefit compared to fixed costs K of undertaking agroforestry. In finance, this is an index that, under certain circumstances, it would be reasonable to expect investors to attempt to maximize. In the agroforestry context, the figure K could represent the value of the land and the labor costs required to run the farm, the farmer having the option of giving up agroforestry and selling the land. If it is assumed that these costs are £7,000 a hectare (£6,000 for the land and £1,000 for the labor allowing for the fact that the time horizon is 40 years) the agroforestry system that maximizes the Sharpe index is that with 300 trees per hectare, which gives an index of 1.17. The best that can be achieved under coarse-level mixing is a 100% allocation of land to sheep. This gives an index of 0.51. Note that the land price is less than the maximum ENPV as this latter has risk associated with it.

The Sharpe index can be shown diagrammatically as a straight line from the point K on the y -axis to the relevant point on the efficient frontier. Maximizing the Sharpe index is equivalent to maximizing the slope of such a line. This is shown in Figure 8 where points S and F are the same as those in Figure 7, point AG is the agroforestry system that maximizes the Sharpe index and M is the point of maximum ENPV.

It follows that the efficient frontier in this example is the line K - AG - M . Any rational, risk-averse farmer would choose systems along that line. The actual point chosen would depend on the risk preferences of the farmer. Diagrammatically, this would be shown by an indifference curve map with the optimum choice being the point on the efficient frontier that coincided with the highest indifference curve. If that point was in segment K - AG the implication would be that the farmer would sell or lease some of the land and put the remainder under the agroforestry system

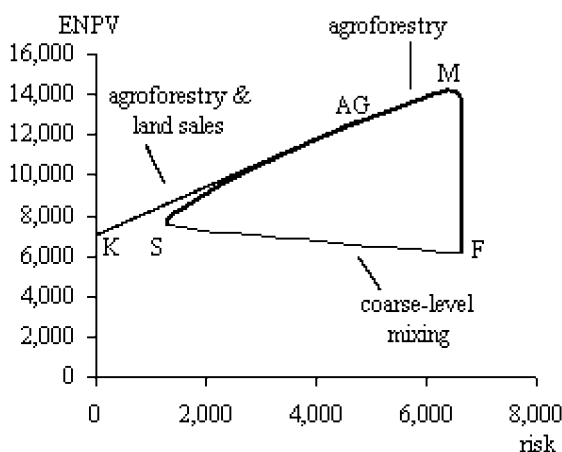


Figure 8. The efficient frontiers for coarse-level mixing and agroforestry with land-sales also allowed.

represented by AG. In the segment AG-M the farmer would opt for an agroforestry system with an initial tree stocking rate of between 300 and 500 stems per hectare. Thus, in this situation, agroforestry rather than coarse mixing of monocultures is always indicated.

In finance the line K-AG can be extended to the right on the assumption of borrowing. Here, such an extension to the line would represent the purchase or renting of extra land and labor at £7,000 a hectare. If such an assumption is allowed the only rational agroforestry system is that shown by AG. Farmers would adjust risk by moving along the extended line K-AG by adjusting the amount of land they farmed. But all farmers would institute system AG on their land.

Altering the assumed correlation terms has virtually no effect on risk in this case. The result is that, for all combinations of correlations between prices and volumes from minus one to plus one, the efficient frontier in Figure 7 remains virtually unaltered. Indeed, no diagram is offered here as, given the scale, the difference is virtually invisible. The reason is that the assumption of serial independence in the price and volume variables means that the only non-zero correlation term relates to year forty. In that year the effect of discounting is at its greatest and so the effect of diversification is minimized. For example, if both the correlations between prices and volumes are plus one, the overall risk of the 500 stems per hectare system is 42,525,259 (as a variance). The covariance element of that is only 1,160,033, or 2.7% of the total risk.

In effect, because of the relatively long rotation of a temperate-zone agroforestry system and the reasonable assumption of economically efficient product markets the diversification effects are very small. Thus, the benefits, or otherwise, of agroforestry derive from the physical/biological effects of intermixing. However, as the following extension shows, this is not the case in shorter rotation systems and, hence, in the application of agroforestry in tropical contexts.

6. A HYPOTHETICAL SHORT-ROTATION AGROFORESTRY SYSTEM

To investigate the effect of shortening the rotation, which according to the discussion above should increase the effects of diversification, the following hypothetical agroforestry system was used. The rotation of the tree crop was reduced to five years and the timber crop in year five was taken to be 60 cubic meters. The value for α for the Gompertz function for yield was, therefore, 60 and not the 480 shown in the table in the Appendix. Thus, a yield class of 12 was still assumed. All of the other variables in the system remained unaltered except for the price of timber. The price assumed was £15 per cubic meter for the current year. In essence, the system can be thought of as a temperate one producing biomass for, say, energy production. Alternatively, it could be viewed as closer to the situation in tropical agroforestry with an annual crop and a short-rotation forest crop.

The result for the efficient frontier is shown in Figure 9 where it is assumed that all correlations are zero. This is a very different frontier from that in Figure 7 describing the previous system. The coarse-level mixture and true agroforestry

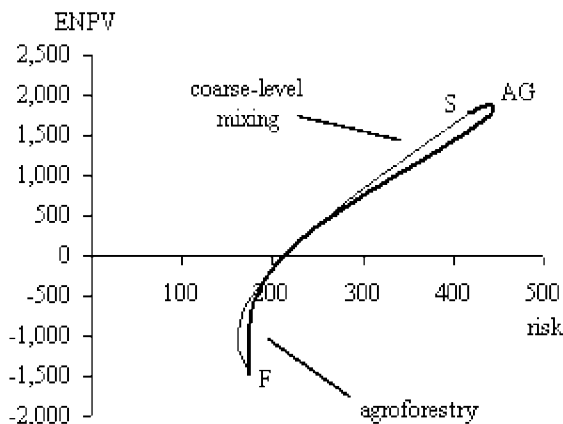


Figure 9. The efficient frontiers for short-rotation sheep/timber agroforestry systems and coarse-level mixing (zero correlation between prices and volumes assumed).

efficient frontiers are nearly coincident and the effect of diversification very apparent. The initial movement in the “top-right” part of the agroforestry frontier, S-AG, reflects the effect of the volume and risk increase that was discussed above, but after that the effect of diversification begins to dominate and the curve is very similar to that shown in the hypothetical example in Figure 1. Note, also, that unlike the frontier in the financial context or the analogous coarse-level mixing frontier, the agroforestry frontier is not necessarily convex from below. The shape depends on the crop interactions.

Although it is not very clear in the diagram, the coarse-level mixing and agroforestry frontiers cross at a positive expected net present value. Thus, for some part of the profitability range coarse-level mixing is better, for some agroforestry dominates.

The result here is interesting because with the price chosen, forestry operates at a loss for planting densities except those between 300 and 500. But despite this apparent lack of profitability, the system with the highest Sharpe index is the agroforestry system using 300 stems to a hectare. Although not clear from the diagram, this is better than any of the coarse-level mixing and better than sheep alone, the slope of the line from the y -axis being maximized at point AG. Note that here, because profitability is lower than the previous case, a value for K of £1,000 per hectare has been used. This is the equivalent of about £4,000 per hectare over 40 years, or eight rotations.

If the assumed correlation coefficients are altered the diagrams change noticeably. Now, the covariance term becomes large enough to influence risk significantly. For example, if both correlations are plus one, the overall risk of a 500 stem per hectare system is 266,103 measured as a variance. The covariance part of

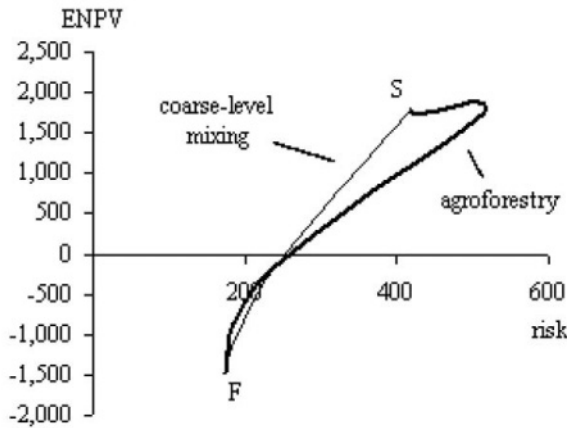


Figure 10. The efficient frontiers for short-rotation sheep/timber agroforestry systems and coarse-level mixing (correlations of plus one assumed between prices and between volumes).

this is 71,139, or 27% of the total. This is ten times greater than the proportion in the forty-year rotation example above.

Figure 10 shows the situation with the worst diversification possibilities. In this case both the correlation of volumes and prices are both taken to be plus one. The agroforestry systems are dominated by the coarse-level mixtures at all positive ENPVs so no agroforestry system would be chosen by a risk-averse rational farmer. The Sharpe index is maximized for the all-sheep option at 1.90.

On the other hand, if the correlation coefficients are set to minus one, the result is shown in the diagram in Figure 11. In this case, the true-agroforestry frontier dominates that of coarse-level mixing at all positive ENPVs and no rational farmer would do anything except intimately mix sheep and trees to gain from the financial and non-financial effects. The Sharpe index is maximized with a tree planting density of around 400 stems at 2.39 indicating that, on that indicator at least, the optimum system would be one that is recognizably agroforestry.

The influence of the correlation coefficients on the relationship of the positions of the two frontiers is shown in Figure 12 below. The y-axis shows the price correlation and the x-axis the volume correlation. At negative correlations of sufficient magnitude agroforestry systems dominate those of coarse-level mixing as shown in Figure 11. At these correlations, agroforestry is the only rational choice. At positive correlations the agroforestry systems are themselves dominated, as in Figure 10, and so no farmer would chose agroforestry over a coarse crop combination.

In the central section of the graph labeled “neither system dominates” the choice will depend on the preferences of the farmer as the frontiers take the form of those shown in Figure 9 with coarse-level mixing dominating in some ranges but not in all.

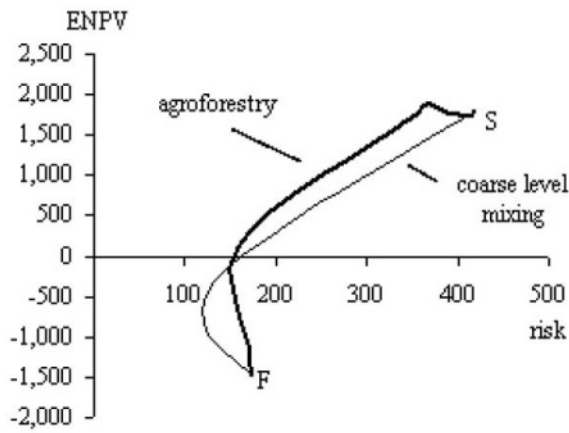


Figure 11. The efficient frontiers for short-rotation sheep/timber agroforestry systems and coarse-level mixing (correlations of minus one assumed between both prices and volumes).

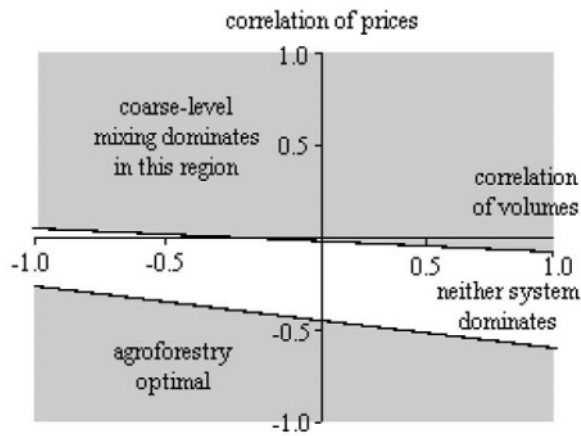


Figure 12. Relationship between optimum land use choice and price and volume correlations.

7. CONCLUDING REMARKS

It should go without saying that the actual results derived here depend upon the figures that have been assumed. Although the actual data used are derived from various sources, many of the interrelationships are speculative. However, the results are intriguing and show that a Markowitz-style analysis of agroforestry adds insights into the analysis of such systems.

A number of points would bear further investigation. For example, are the following statements generally true or are they the results of the specific example chosen here?

- The effects of coarse-level diversification are likely to be very small in temperate-zone agroforestry systems if the rotation length of the forestry element of the system is “long” or the discount rate is high.
- In temperate-zone agroforestry the interactions of the different crop elements are critical to the desirability or otherwise of agroforestry, and price diversification effects can be ignored.
- For shorter rotation crops the correlations of price and volume are critical in both the design and choice of an agroforestry system.
- Paradoxically, therefore, in tropical/subtropical situations it is likely that data for both crop and price interactions are needed – maximum data requirements appear to occur in situations with minimum data availability. This is an observation that is probably true for all operations research-style approaches (Betters, 1988).

Not only are data a problem for tropical/subtropical situations. Poor rural households may approach the issue of risk differently than temperate richer landowners. Here, the analysis assumes that choices can be modeled by smooth indifference curves which, usually by implicit assumption, are sufficiently “flat” to allow the conclusion that optimum solutions will lie somewhere along the efficient frontier and not at a border solution. Thus, it is implicit when discussing diagrams such as Figure 7, that the choice of an optimum system lies somewhere between *S* and the maximum point on the curve. This implies that, while risk is seen as undesirable, it is not catastrophically bad. However, in the tropical arena households are often close to subsistence level. The implication would be that avoiding bad outcomes is a question of survival rather than just a trade-off against the possibility of a higher return.

Technically, the indifference curve map that is implicit in such situations would be represented in the diagrams above by lines that were nearly vertical, showing that a very slight increase in risk would have to be compensated for by a very high increase in potential income.

Thus, in the case of poor land owners, it is possible that corner solutions would become more common. In portfolio terms, and especially in terms of Figure 1, this would seem to mean that monocultures would be indicated unless the efficient frontier “bent back” on itself as it did in the illustration in Figure 1.

However, in such contexts where subsistence farmers are producing crops for their own consumption rather than for the market, decisions are likely to be made in terms of nutritional variables rather than monetary ones. In this case, the relevant figures in the analysis become the expected nutritional output and its variance. Thus, monocultures would be somewhat rarer and the points on the efficient frontier could represent outputs measured in a single nutritional variable that is the result of a mixture of crops.

While this does not invalidate the ideas here it does mean that there are likely to be fruitful areas for work. Even for farmers marketing their crops, ideas of catastrophic risk can be incorporated. For example, there are a number of so-called “safety-first” models that have been proposed in finance. The interested reader is referred, again, to Elton and Gruber’s text (1995) where three such models are outlined. One, Roy’s criterion, would translate here to minimizing the probability that the ENPV of the agroforestry system was below some predefined subsistence level. Katoka’s criterion maximizes the lower limit of the ENPV range subject to the constraint that the probability of achieving this lower limit is not greater than some predetermined value. A third criterion, due to Telser, suggests maximizing the ENPV subject to the constraint that the probability of the actual ENPV being less the subsistence level is not greater than some predetermined figure.

In the context of finance it can be shown that such criteria often lead to the same result as conventional Markowitz-type analysis. However, there is another factor that differentiates subsistence level farming from western finance. In the usual application of these models, the returns from financial assets are symmetrically distributed. Thus, the idea of skewness can conveniently be ignored. However, in many tropical situations it is likely that agricultural outcomes are very skewed and this skewness is likely to be an important decision factor. Thus, in “normal” years output varies according to a usual symmetrical type of distribution but, every now and then, the harvest could be catastrophically low. This downside risk (along with the downside covariances and co-skewnesses) could well figure in the decision-making in subsistence farming but is ignored in the analysis here.

Despite, or maybe because of, these provisos, the portfolio theory approach adds extra insights to agroforestry and can be used in tandem with biologically-based research to define optimum agroforestry systems. If risk in agroforestry is more fully understood and incorporated into evaluations, it is likely that agroforestry will be more attractive to farmers and hence enjoy a higher uptake.

8. APPENDIX

In the derivation of the equations in this chapter, use is made of the following properties of random numbers. First, the expected value of a sum of random numbers is equal to the sum of their expected values. Using the notation $E(x)$ for the expected value of the random variable x , the first result can be written as equation A1.

$$E(x + y) = E(x) + E(y) \quad (\text{A1})$$

If x and y are not correlated, the expected value of the product is equal to the product of the expected values. In other words,

$$E(x \times y) = E(x) \times E(y) \quad (\text{A2})$$

The variance of the sum of two random numbers is one of the principal equations in portfolio theory and is given in equation A3.

$$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{x,y} \quad (\text{A3})$$

The following results are also employed and can be found in any standard statistical textbook. If n is a constant (or at least is not a stochastic variable) then the following are true.

$$E(nx) = nE(x) \quad (\text{A4})$$

$$\sigma_{nx}^2 = n^2 \sigma_x^2 \quad (\text{A5})$$

$$\sigma_{mx,ny} = mn\sigma_{xy} \quad (\text{A6})$$

The basic price and volume model assumes that the random movement can be modeled as a growth element. Thus, if P_t is the actual price it can be defined in equation A7 where ε is a normally distributed random variable with a mean of zero and variance of σ^2 .

$$P_t = E(P_t)(1 + \varepsilon) \quad (\text{A7})$$

A similar equation can be written for the volume term. Note that the assumed growth in prices given by the factor μ is included in the expected price for the next period and not in the error term. Thus, $E(P_2) = E(P_1)(1 + \mu)$.

Using equations A2 and A4 above, the expected net present value of forestry, $E(NPV_f)$ can be written as the following where the subscript f refers to forestry, v to volume and p to price, C is the cost that is non-stochastic but varies with the planting density, r is the discount rate and R the rotation. The expected price, $E(P_{f,R})$ is the price of timber in period R the expectation being formed in the current period. This is given by equation 6 in the main text.

$$E(NPV_f) = \frac{E(P_{f,R})E(V_f)}{(1+r)^R} - C_f \quad (\text{A8})$$

The variance of the expected net present value of forestry is somewhat less elegant. The basic form is shown in equation A10 below.

$$\sigma_f^2 = E \left[\left(E(NPV_f) - \frac{E(P_{f,R})(1+\varepsilon_p)E(V_f)(1+\varepsilon_v)}{(1+r)^R} + C_f \right)^2 \right] \quad (A9)$$

If this is expanded and, using the fact that $\varepsilon_v = \varepsilon_p = 0$, the result is given by equation A11.

$$\sigma_f^2 = \frac{E(P_{f,R})^2 E(V_f)^2}{(1+r)^{2R}} (\sigma_v^2 + \sigma_p^2 + \sigma_v^2 \sigma_p^2) \quad (A10)$$

The variance of the error term in price, σ_p^2 is not the same as that given in the text in equation 7. That equation relates to the variance of price itself. Here the variance refers to the multiplicative error term and will be given by $\sigma^2(R)^{0.5}$ where the variance σ^2 is that to be found in Table 2. This is because of the growth in price and increasing variance that this implies. The volume figure is treated differently and so the variance here will be that found in Table 3.

For sheep the expected net present value for production in period t is given by equation A12. This differs slightly from A9 because the cost figure is on a per lamb basis. Higher volume output is likely to lead to more costs for feed, etc. and this is not likely to be the case with extra volume growth in forestry. Also, the volume figure is given by the stocking rate for ewes, taken to be a constant, multiplied by the fertility, $N_s E(F_s)$. It is this latter term to which stochastic variation is attached.

$$E(NPV_s) = \frac{[E(P_{s,t}) - C_s]E(V_s)}{(1+r)^t} \quad (A11)$$

The variance of the net present value for sheep for period t is found by the expansion of equation A13 given in A14.

$$\sigma_s^2 = E \left[\left(E(NPV_s) - \frac{[E(P_{s,t})(1+\varepsilon_p) - C_s]E(V_s)(1+\varepsilon_v)}{(1+r)^t} \right)^2 \right] \quad (A12)$$

$$\sigma_s^2 = \frac{E(V_s)^2}{(1+r)^{2t}} \left(E(P_{s,t})^2 (\sigma_v^2 + \sigma_p^2 + \sigma_v^2 \sigma_p^2) - 2E(P_{s,t})C_s \sigma_v^2 + C_s^2 \sigma_v^2 \right) \quad (A13)$$

Finally, bearing in mind that serial correlation terms are all zero, the only covariance term is that between the final lamb and timber crops. This is given by equation A15 and its simplification in A16 where $\sigma_{vf,a}$ refers to the covariance of the volume term for forestry and agriculture, etc.

$$\sigma_{s,f} = E \left[\left(E(NPV_s) - \frac{E(P_{s,R})(1 + \varepsilon_{p,s}) - C_s}{(1+r)^R} E(V_s)(1 + \varepsilon_{v,s}) \right) \left(E(NPV_f) - \frac{E(P_{f,R})(1 + \varepsilon_{p,f}) E(V_f)(1 + \varepsilon_{v,f})}{(1+r)^R} + C_f \right) \right] \quad (A14)$$

$$\sigma_{s,f} = \frac{E(V_s)E(V_f)E(P_{f,R})}{(1+r)^{2R}} (E(P_{s,R})(\sigma_{vs,f} + \sigma_{ps,f} + \sigma_{ps,f}\sigma_{vs,f}) - C_s\sigma_{vs,f}) \quad (A15)$$

In the calculations used for the main text, the covariance terms within A16 were calculated using the relationship between standard deviations and correlation coefficients.

Much use is made of the Gompertz function in relating variables in the model. The basic function is given in equation A17 below.

$$y = \alpha e^{(-e^{-\beta(x-\chi)})} \quad (A16)$$

In the formula, α represents the limit to which y tends as x increases, β is a slope variable which defines the rate at which the function “moves” from its lower value to its upper and χ is the point of inflection. The values used in this function in its various guises are given in the table below.

Table A1. The parameters of the Gompertz functions used.

| meaning/value | | | |
|---------------|--------------------|--------------------|---|
| y | cumulative volume | initial flock size | lamb output as a percentage of output in year one |
| x | tree stocking rate | tree stocking rate | year |
| α | 480 | 12 | 0.75 |
| β | 0.1 | 0.005 | 0.3 |
| χ | 200 | 800 | See below |

The value for χ in the function defining the decay in lamb output was made dependent upon the initial stocking level for the tree part of the system. The argument is that, at higher stocking levels crown cover occurs sooner and so the point at which pasture production begins to decline will also be sooner. The function used is that given below.

$$\chi = \frac{400}{\sqrt{\text{initial tree stocking rate}}} \quad (\text{A17})$$

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